Processes of freezing and thawing of porous media

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Motivation

- Study the effects of climate and temperature change on soil grounds (Žák A.)

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  - e.g. improve ground structure design and maintenance in cold regions and in winter

For $x \geq 0$ and $t > 0$:

$$\rho C_p \frac{\partial T}{\partial t}(x,t) = \nabla(\kappa \nabla T(x,t)) + Q \quad \text{(heat conduction equation)}$$

$$\kappa \frac{\partial T}{\partial \eta}(x,t)\Big|_S - \kappa \frac{\partial T}{\partial \eta}(x,t)\Big|_L = Lv \quad \text{(Stefan condition)}$$

$$T(x,0) = T_0 \quad \text{(boundary condition)}$$

$$T(0,t) = T(t_0) \quad \text{(initial condition)}$$

where

ρ : density

C_p : heat capacity

κ : thermal conductivity

Q : heat source

L : latent heat

- No phase change - Internal energy

$$U(x,t) = \int_{T*}^{T(x,t)} \rho C_p \, dT$$

- No phase change - Internal energy

$$U(x,t) = \int_{T^*}^{T(x,t)} \rho C_p \, dT$$

- Phase change - Enthalpy

$$H(x,t) = \underbrace{U(x,t)}_{\text{internal energy}} + L \cdot \mathcal{H}(T(x,t))$$

where

L : latent heat

$\mathcal{H}$: Heaviside function

$$\mathcal{H}(T(x,t)) = \begin{cases} 1 & \text{for} \quad T(x,t) > T^* \\ 0 & \text{for} \quad T(x,t) < T^* \end{cases}$$

Heat conduction equation

$$C_p \frac{\partial T}{\partial t}(x,t) + L \frac{\partial \theta}{\partial t}(T(x,t)) = \nabla(\kappa \nabla T(x,t))$$

where

$$\eta : \text{porosity}$$

$$\theta(T(x,t)) = \eta \phi(T(x,t))$$

$$\phi(T(x,t)) = \begin{cases} 1 & \text{if} \quad T(x,t) \geq T^* \\ \left| \dfrac{T^*}{T(x,t)} \right|^b & \text{if} \quad T(x,t) < T^* \end{cases}$$

$$b : \text{positive constant}$$

Ω_S	Domain containing soil
Ω_L	Domain containing liquid (water here)

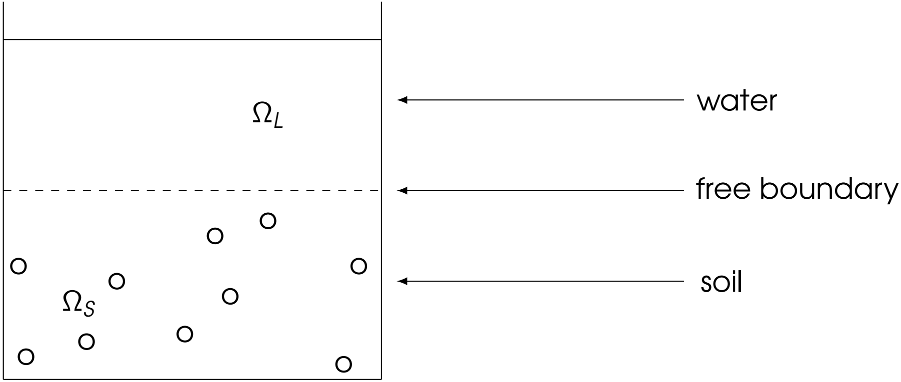

Figure: Illustration of the problem

First derivative of the enthalpy in respect to time :

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$$\begin{aligned}
\frac{\partial H}{\partial t}(x,t) &= \frac{\partial U}{\partial t}(x,t) + L \cdot \frac{\partial \mathcal{H}}{\partial T}(T(x,t)) \\
&= \rho C_p \frac{\partial T}{\partial t}(x,t) + L \cdot \frac{\partial \mathcal{H}}{\partial T}(T(x,t)) \\
&= \rho C_p \frac{\partial T}{\partial t}(x,t) + L \cdot \frac{\partial \mathcal{H}}{\partial T}(T(x,t) - T^*)\frac{\partial T}{\partial t}(x,t) \\
&= \nabla(\kappa \nabla T(x,t)) + Q
\end{aligned}$$

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\end{aligned}$$

Regularization :

First derivative of the enthalpy in respect to time :

$$
\begin{aligned}
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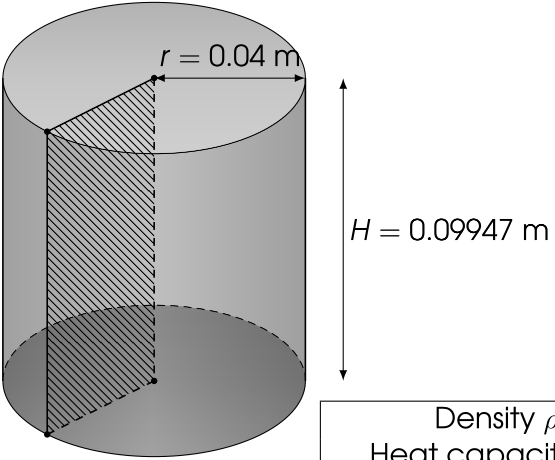
&= \nabla(\kappa \nabla T(x,t)) + Q
\end{aligned}
$$

Regularization :

$$
\begin{aligned}
\frac{\partial U}{\partial t}(x,t) &= \rho C_p \frac{\partial T}{\partial t}(x,t) \\
&= \nabla(\kappa \nabla T(x,t)) - L \cdot \frac{\partial \mathcal{H}_\varepsilon}{\partial T}(T(x,t) - T^*)\frac{\partial T}{\partial t}(x,t)
\end{aligned}
$$

Freezing of pure water H_20

Modelling



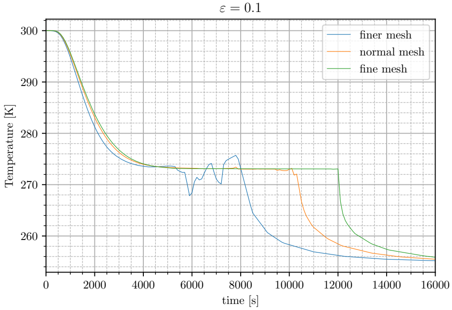
$V = 0.5$ L

Density ρ	997 kg/m^3
Heat capacity C_p	4200 J/kg·K
Thermal conductivity κ	0.5918 W/m·K

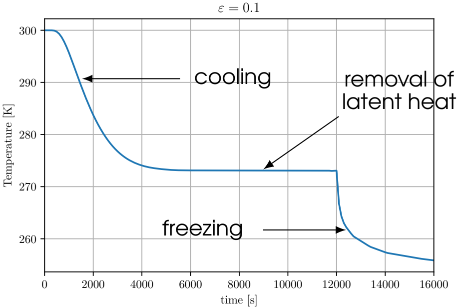
Finer mesh animation : `https://youtu.be/oWznctWiYmo`

Freezing of pure water H_20

Modelling



(a) Freezing evolution with different types of meshes



(b) Freezing evolution with finer mesh

Figure: Freezing evolution of water

Freezing of porous media

Experiment

(a) Freezing and thawing experiment with smaller diameter grains of pink sandstone

(b) Freezing and thawing experiment with larger diameter grains of pink sandstone

Freezing of porous media
Experiment

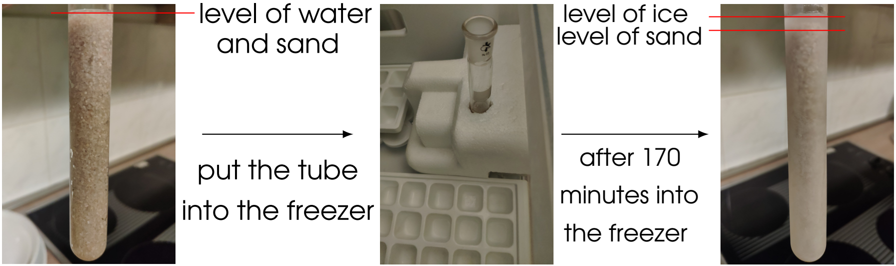
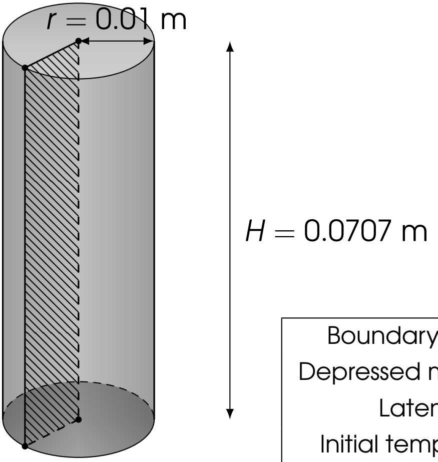


Figure: Experimental setup



$V = 22.2$ mL

Boundary condition (T_0)	-0.1 °C
Depressed melting point (T^*)	-0.01 °C
Latent heat (L)	$2.5 \cdot 10^6$ J/kg
Initial temperature ($T(t_0)$)	0.2 °C
Thermal conductivity (κ)	2.2 W/m·°C
Heat capacity (C_p)	$4.2 \cdot 10^6$ J/kg·°C

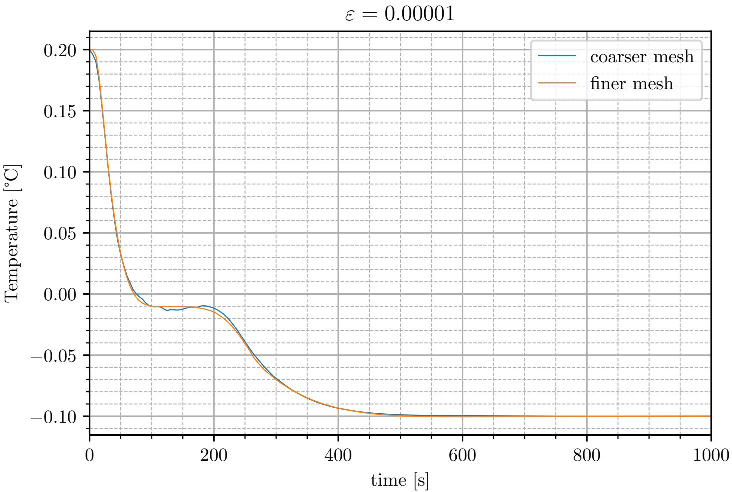


Figure: Freezing evolution of wet sand with different types of meshes

Freezing of water with gas

Is the gas released from ice ?

- Frozen Perrier water animation :
  `https://youtu.be/SeN-wov-Jsw`
- Thawed Perrier water animation :
  `https://youtu.be/cFtkSg1dRAY`

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